DETAILED CONTENTS OF TOPICS

NUMBER PATTERNS

You will find that patterns occur in almost every topic in maths! They often help us to make sense of things and, most of all, we learn to think!

The **National Curriculum Statement** says of **Patterns** . . .

> ***"The mastery of maths depends on being able to:***
> ***investigate/explain/recognise/extend patterns;***
> ***formulate conjectures ("have a guess at how it works");***
> ***prove or disprove your conjecture;***
> ***generate a "rule" i.e. generalise;***
> ***see links within and across topics and real life."***

Patterns (or sequences) are fun and we will cover a wide range of examples. Try to do these on your own as much as possible. Various approaches will, however, also be suggested to help you get the idea. Enjoy the challenge!

❑ GUIDED INVESTIGATION 1

- encouraging recognition of standard sequences !

Some examples - try these . . .

(1) 2 ; 4 ; 6 ; 8 ; . . .

(2) 3 ; 6 ; 9 ; 12 ; . . .

(3) 2 ; 4 ; 8 ; 16 ; . . .

(4) 1 ; 4 ; 9 ; 16 ; . . .

These are "sequences" of numbers and we refer to the numbers as **TERMS** and use the symbol **T** as follows:

$\mathbf{T_1}$ = 2 means : the first term is 2

$\mathbf{T_2}$ = 4 means: the second term is 4

In each case, say what

(a) the **next number** would be, (b) the **tenth**, (c) the **100th** and (d) the **nth** ?

Answers to Guided Investigation 1

	(a) Next number	(b) 10^{th} number	(c) 100^{th} number	(d) n^{th} number
(1)	10	2(10) = 20	2(100) = 200	$2n$
(2)	15	3(10) = 30	3(100) = 300	$3n$
(3)	32 (= 2^5)	2^{10} (= 1 024)	2^{100} (= 1,26 × 10^{30}!)	2^n
(4)	25	10^2 (= 100)	100^2 (= 10 000)	n^2

How have you done?

Do you see how necessary it is to think? You can develop the skills to analyse, predict and generalise. So, let's discuss some possible approaches . . .

Discussion

There was more than one way to see what the next number would be . . . .

If you said: We just add 2 each time, that was fine, but, if you *recognise* that this is the set of *even numbers*, and that

the **1st** number is $2 \times \mathbf{1}$, i.e. $T_1 = 2(\mathbf{1})$;

the **2nd** number is $2 \times \mathbf{2}$, i.e. $T_2 = 2(\mathbf{2})$;

the **3rd** number is $2 \times \mathbf{3}$, i.e. $T_3 = 2(\mathbf{3})$

then, you could say: the **5th** number is $2 \times \mathbf{5}$, i.e. $T_5 = 2(\mathbf{5})$

the **10th** number is $2 \times \mathbf{10}$, i.e. $T_{10} = 2(\mathbf{10})$

the **100th** number is $2 \times \mathbf{100}$, i.e. $T_{100} = 2(\mathbf{100})$

In general: ∴ the **nth** number is $2 \times \mathbf{n}$, i.e. $T_n = 2\mathbf{n}$

> **2n** is the GENERAL TERM for the set of EVEN numbers
> or, we say **2n** is the STANDARD FORM of an EVEN number
> (because an even number is always **2 × a natural number**).

Imagine a row of lockers numbered 1; 2; 3 . . .

Let's place the even numbers inside:

n ⇨	**1**	**2**	**3**	**4**	**5**	.....	**10**	.....	**100**	.....	**n**	.....
2n ⇨	**2**	**4**	**6**	**8**	**10**	.....	**20**	.....	**200**	.....	**2n**	.....

The term (number) in the locker depends entirely on **the locker number**, not so?

If the locker number is **n,** the term is $\mathbf{T_n}$

and, in this case, $\mathbf{T_n = 2n}$

Repeat the 'discussion' above for : the MULTIPLES of 3 in e.g. (2)

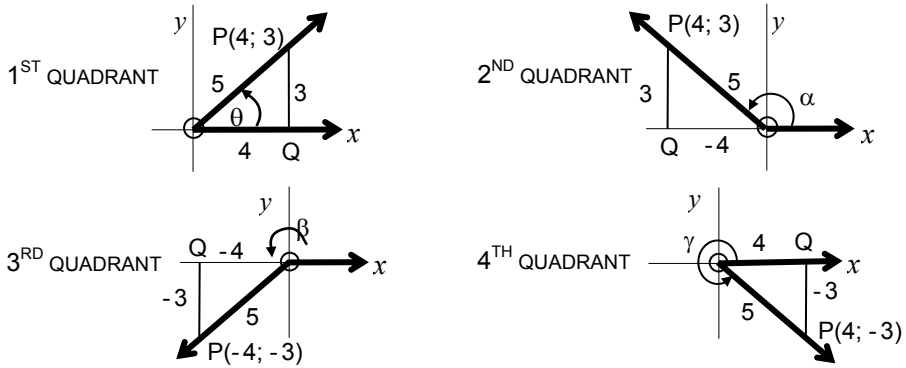
the POWERS of 2 in e.g. (3)

the SQUARES of natural numbers in e.g. (4)

The final GENERALISATIONS ("rules"):

(1) $T_n = 2n$ (2) $T_n = 3n$ (3) $T_n = 2^n$ (4) $T_n = n^2$

☐ SUMMARY OF OUR FINDINGS



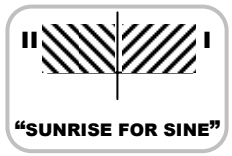
1ST QUADRANT	2ND QUADRANT	3RD QUADRANT	4TH QUADRANT
$\sin\theta = \frac{3}{5}$	$\sin\alpha = \frac{3}{5}$	$\sin\beta = -\frac{3}{5}$	$\sin\gamma = -\frac{3}{5}$
$\cos\theta = \frac{4}{5}$	$\cos\alpha = -\frac{4}{5}$	$\cos\beta = -\frac{4}{5}$	$\cos\gamma = \frac{4}{5}$
$\tan\theta = \frac{3}{4}$	$\tan\alpha = -\frac{3}{4}$	$\tan\beta = \frac{3}{4}$	$\tan\gamma = -\frac{3}{4}$

Examine these results and use them to know and understand . . . .

⟐ SIGNS of your trig ratios IN A FLASH!

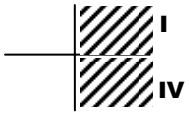
$\sin\theta = \frac{y}{r}$ ← and **y is positive** in **I** and **II**

∴ **sin θ is POSITIVE** in quadrants 1 & 2
(and negative in 3 & 4)



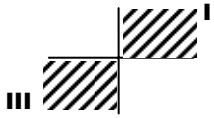
$\cos\theta = \frac{x}{r}$ ← and **x is positive** in **I** and **IV**

∴ **cos θ is POSITIVE** in quadrants 1 & 4
(and negative in 2 & 3)



$\tan\theta = \frac{y}{x}$ ← ← and **x & y** have the same sign in **I** and **III**

∴ **tan θ is POSITIVE** in quadrants 1 & 3
(and negative in 2 & 4)

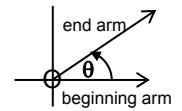


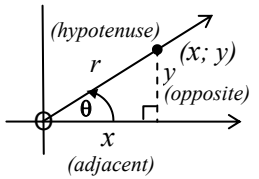
Apparently this is **THE BMW SYMBOL**!

It is really easy to get these PICTURES into your head so that you know the SIGNS of your trig ratios IN A FLASH ! NO MORE CAST RULE !!!

How to find the trig ratios of ANY angle: 4 STEPS !

1. **Place the L in STANDARD POSITION** (starting at $\overrightarrow{OX}$) . . .
2. **Pick a point (x; y) on the 'end arm' of the L**
   - we'll call its distance from the origin **r**
3. **Write down x = y = r =**
4. **Apply the DEFINITIONS**

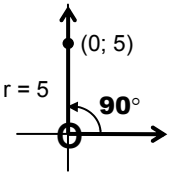




$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

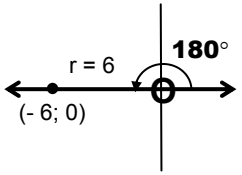
⟐ The Trig ratios of multiples of 90°

Use this procedure to find the trig ratios of 0° ; 90° ; 180° ; 270° & 360°



$\sin 90° = \frac{y}{r} = \frac{5}{5} = 1$
$\cos 90° = \frac{x}{r} = \frac{0}{5} = 0$
$\tan 90° = \frac{y}{x} = \frac{5}{0} = \infty$

$x = 0$; $y = 5$; $r = 5$



$\sin 180° = \frac{y}{r} = \frac{0}{6} = 0$
$\cos 180° = \frac{x}{r} = \frac{-6}{6} = -1$
$\tan 90° = \frac{y}{x} = \frac{0}{-6} = 0$

$x = -6$; $y = 0$; $r = 6$

Do 0°, 270° and 360° too. Check your answers below.

SUMMARY

θ:	0°	→	90°	→	180°	→	270°	→	360°
sin θ :	0	- - - →	1	- - - →	0	- - - →	- 1	- - - →	0
cos θ :	1	- - - →	0	- - - →	- 1	- - - →	0	- - - →	1
tan θ :	0	- - - →	∝	- - - →	0	- - - →	± ∝	- - - →	0

❒ **As θ : 0° → 360°**