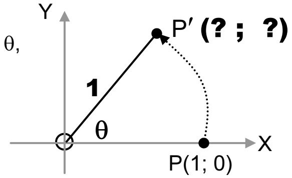
COMPOUND ANGLE FORMULAE & ROTATION OF A POINT THROUGH AN ANGLE, θ

QUESTIONS

COMPOUND ANGLES FORMULAE

1.1 If point P is rotated about the origin through an angle θ, determine the coordinates of the image P' of P in terms of θ.

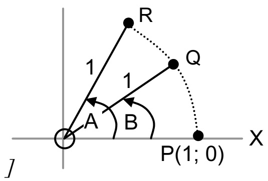


1.2 In the figure alongside

OR = OQ = OP = 1 unit

$R\hat{O}X = \hat{A}$ and $Q\hat{O}X = \hat{B}$

[i.e. P has been rotated, first through $\hat{A}$ to R, then (from its original position) through $\hat{B}$ to Q.]



1.2.1 Write down the coordinates of R and Q in terms of A and B, respectively.

1.2.2 $R\hat{O}Q$ = ... *(in terms of $\hat{A}$ and $\hat{B}$)*

1.2.3 By using the **cosine rule** in ΔROQ, find an expression for $\mathbf{RQ^2}$ in terms of $\hat{A}$ & $\hat{B}$.

1.2.4 By using the **distance formula** (Coordinate Geometry), find an expression for $\mathbf{RQ^2}$ (again !) in terms of $\hat{A}$ & $\hat{B}$.

1.2.5 Combine your results of 1.2.3 and 1.2.4 and deduce that :

cos(A - B) = cos A cos B + sin A sin B ?

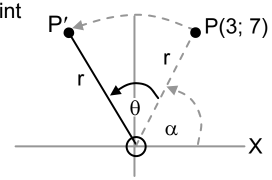
1.3.1 Derive a formula for **cos (A + B)** from the formula for cos(A - B) in 1.2.5.

1.3.2 Derive a formula for **sin (A + B)** from the formula for cos(A - B) in 1.2.5.

ROTATION OF A POINT THROUGH θ

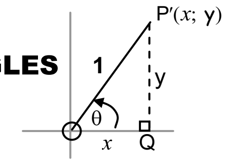
2.1 If point P (3; 7) in the figure alongside is translated through an angle θ about the origin, to point P', find the coordinates of P' in terms of θ.

2.2 Now find the coordinates of P' if $\theta = 52°$.



ANSWERS

COMPOUND ANGLES



1.1 NOTE : ***OP = 1 unit !***

Let P' be $(x; y)$

Draw PQ $\perp$ x-axis

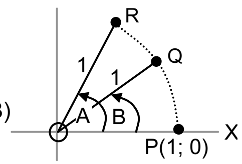
Clearly : $\frac{x}{1} = \cos\theta$ and $\frac{y}{1} = \sin\theta$

i.e. $x = \cos\theta$ and $y = \sin\theta$

i.e. ***P′ is the point*** **(cos θ; sin θ)** ◄

1.2.1 **R(cos A ; sin A)** & **Q(cos B ; sin B)** ◄

1.2.2 $R\hat{O}Q = \hat{A} - \hat{B}$ ◄



1.2.3 **COSINE RULE :**

$\mathbf{RQ^2} = 1^2 + 1^2 - 2(1)(1)\cos(A - B)$

$= 2 - 2\cos(A - B)$ ◄

1.2.4 $\mathbf{RQ^2} = (\cos A - \cos B)^2 + (\sin A - \sin B)^2$ . . . **DISTANCE FORMULA**

$= \cos^2 A - 2\cos A\cos B + \cos^2 B + \sin^2 A - 2\sin A\sin B + \sin^2 B$

$= 2 - 2\cos A\cos B - 2\sin A\sin B$ ◄ *. . . $\sin^2\theta + \cos^2\theta = 1$*

1.2.5 ***Equating the two expressions for RQ^2 in 1.2.3 & 1.2.4 :***

$\therefore$ $2 - 2\cos(A - B) = 2 - 2\cos A\cos B - 2\sin A\sin B$ ◄

Subtract 2 :

$\therefore$ $-2\cos(A - B) = -2\cos A\cos B - 2\sin A\sin B$

Divide by -2 $\left(\textit{or multiply by } -\frac{1}{2}\right)$

$\therefore$ **cos(A - B) = cos A cos B + sin A sin B** ◄

1.3.1 **cos (A + B)** = cos [A - (- B)]

*i.e. The cos of the **difference** of 2 angles !*

= cos A cos (- B) + sin A sin (- B)

= cos A cos B + sin A (- sin B)

= cos A cos B - sin A sin B ◄

1.3.2 **sin (A + B)** = cos [90° - (A + B)]

= cos [90° - A - B]

*Again, the cos of the **difference** of 2 angles*

= cos [(90° - A) - B]

= cos (90° - A) cos B + sin (90° - A) sin B

= sin A cos B + cos A sin B ◄

ROTATION THROUGH θ

2.1 Say $P\hat{O}X = \alpha$

then $\frac{3}{r} = \cos\alpha$ & $\frac{7}{r} = \sin\alpha$

$\therefore$ $3 = r\cos\alpha$ $\therefore$ $7 = r\sin\alpha$

Similarly, if P' is the point (a; b) :

$\frac{a}{r} = \cos(\alpha + \theta)$ & $\frac{b}{r} = \sin(\alpha + \theta)$

$\therefore$ $\mathbf{a} = r\cos(\alpha + \theta)$ $\therefore$ $\mathbf{b} = r\sin(\alpha + \theta)$

$= r[\cos\alpha\cos\theta - \sin\alpha\sin\theta]$ $= r[\sin\alpha\cos\theta + \cos\alpha\sin\theta]$

$= \underbrace{r\cos\alpha}\cos\theta - \underbrace{r\sin\alpha}\sin\theta$ $= \underbrace{r\sin\alpha}\cos\theta + \underbrace{r\cos\alpha}\sin\theta$

$\mathbf{= 3\cos\theta - 7\sin\theta}$ $\mathbf{= 7\cos\theta + 3\sin\theta}$

$\therefore$ **P′ (3cos θ - 7sin θ ; 7cos θ + 3sin θ)** ◄

2.2 **P′(-3,67 ; 6,67)** ◄