



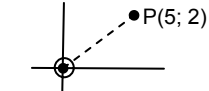
TRANSFORMATIONS

(Gr. 11 & 12)

Transformations, especially translations and reflections, have occurred in earlier sections - functions, coordinate geometry and trigonometry. This section offers translations, reflections, enlargements and rotations (at first, just through $\pm 180^\circ$ or $\pm 90^\circ$, as for Grade 11).

1.1 Write down the image of $P(5; 2)$ and $Q(x; y)$ - any point on the Cartesian plane - under the following transformations:

- (1) translation 2 units right and 3 units down.
- (2) reflection in the Y-axis.
- (3) reflection in the X-axis.
- (4) reflection in the line $y = x$.
- (5) reflection in the line $y = -x$.
- (6) reflection in the origin.
- (7) rotation through 180° (anti-clockwise) about the origin.
- (8) rotation through 180° (clockwise) about the origin.
- (9) rotation through 90° (anti-clockwise) about the origin.
- (10) rotation through 90° (clockwise) about the origin.
- (11) enlargement of OP & OQ by a factor of 3 through the origin.
- (12) enlargement of OP & OQ by a factor of $\frac{1}{3}$ through the origin.
- (13) reduction of OP & OQ by a factor of 3 through the origin.
- (14) glide reflection: 6 units right and reflection in the x -axis.

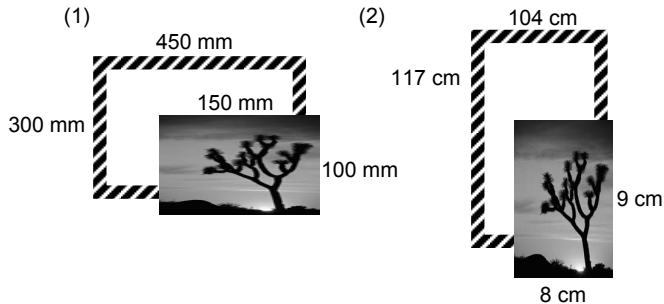


1.2 Under which of these transformations would the image of a triangle, say, be similar (and not necessarily congruent)?

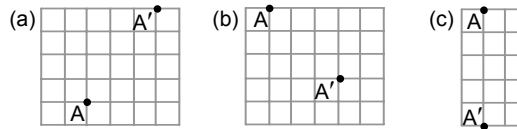
2. Describe the transformations in words that would transform the point $(3; 4)$ to:

- (1) $(4; 3)$ (2) $(-3; -4)$ (3) $(5; -1)$ (4) $(-4; 3)$
 (5) $(-4; -3)$ (6) $(3; -4)$ (7) $(4; -3)$ (8) $(-3; 4)$

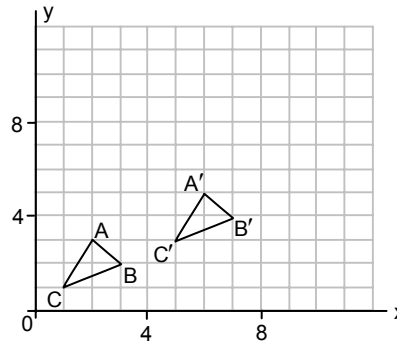
3.1 Find the scale factor of enlargement needed to fit each of these photographs into their new frames.



3.2 In each of the following, write the translation which takes A to A' in terms of x and y if A is the point $(x; y)$.



4.



4.1 In the sketch above, describe in words the transformation, T_1 , from $\triangle ABC$ to $\triangle A'B'C'$.

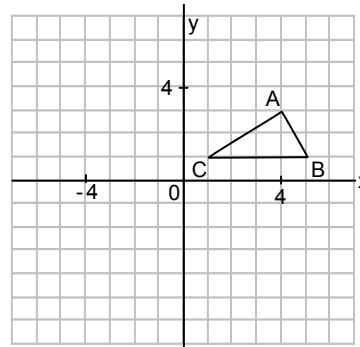
4.2 T_2 is an enlargement through the origin by a factor of 2. Perform this transformation on $\triangle ABC$ to give $\triangle A''B''C''$. Write down the coordinates of the vertices of this triangle.

4.3 Write down the images of a point $(x; y)$ under transformations, T_1 and T_2 , in terms of x and y .

4.4 Use the symbols \equiv or \parallel to relate:

- (1) $\triangle ABC \dots \triangle A'B'C'$ (2) $\triangle ABC \dots \triangle A''B''C''$

5.



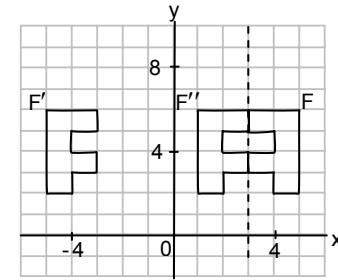
5.1 On the grid above, rotate $\triangle ABC$ 90° clockwise about the origin. Label the image of this rotation $\triangle A'B'C'$.

5.2 Rotate $\triangle A'B'C'$ 180° about the origin and label this image $\triangle A''B''C''$.

5.3 What single transformation has the same effect as the combination of these two rotations?

5.4 Give an alternative for the 'inverse' of this combined transformation, i.e. the transformation of $\triangle A''B''C''$ to $\triangle ABC$.

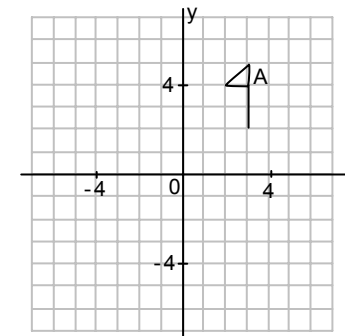
6.



Describe the transformations:

6.1 F to F' 6.2 F' to F'' in words and in terms of x and y .

7.

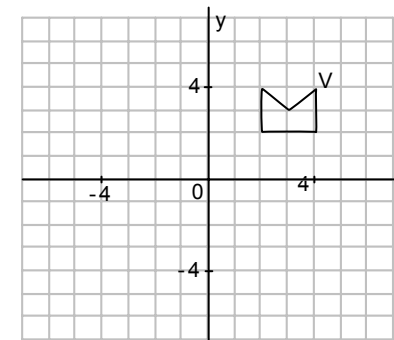


7.1 Reflect the figure A in the y -axis and label the reflection A' .

7.2 Reflect A' in the line $y = x$ and label this reflection A'' .

7.3 Find a single rotation which would take A to A'' .

8.



8.1 The figure V has to be enlarged through the origin by a factor of $\frac{1}{2}$ (i.e. reduced by a factor of 2).

8.1.1 Use the grid to draw this enlargement, V' .

8.1.2 If the area of V is p square units, what is the area of V' ?

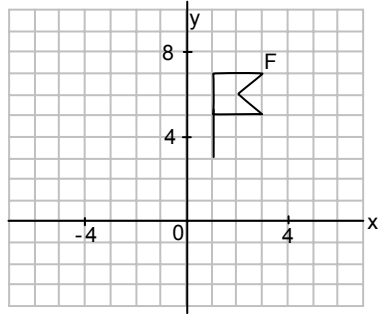
8.2 Figure V is rotated 180° through the origin.

8.2.1 Give the general rule (in terms of x and y) of a point undergoing this transformation.

8.2.2 Draw the image V'' of this transformation.



9.



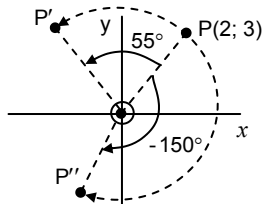
- 9.1 Flag F on the grid above is rotated 90° anticlockwise about the origin. Use the grid to draw the image F' of this rotation.
- 9.2 Draw F'' , the reflection of F' in the y -axis, on the same grid.

GRADE 12 only**Rotation of a point through any angle θ** **NOTE**

It is essential to work through P1.36 column 3 to understand the theory behind the method in Question 10 here.

(For your convenience the answers have been provided beneath the questions.)

10. Calculate the coordinates of the image P' of the point $P(2; 3)$ after rotation about the origin, through an angle of



- 10.1 55° (answer to 2 decimal places).
- 10.2 -150° (in simplest surd form).



TRANSFORMATIONS

- 1.1 (1) $(7; -1) ; (x+2; y-3)$ (2) $(-5; 2) ; (-x; y)$
 (3) $(5; -2) ; (x; -y)$ (4) $(2; 5) ; (y; x)$
 (5) $(-2; -5) ; (-y; -x)$ (6) $(-5; -2) ; (-x; -y)$
 (7) $(-5; -2) ; (-x; -y)$ (8) $(-5; -2) ; (-x; -y)$
 (9) $(-2; 5) ; (-y; x)$ (10) $(2; -5) ; (y; -x)$
 (11) $(15; 6) ; (3x; 3y)$ (12) $\left(\frac{5}{3}; \frac{2}{3}\right) ; \left(\frac{x}{3}; \frac{y}{3}\right)$
 (13) $\left(\frac{5}{3}; \frac{2}{3}\right) ; \left(\frac{x}{3}; \frac{y}{3}\right)$ (14) $(11; -2) ; (x+6; -y)$

1.2 (11), (12) and (13)

2. (1) reflection in the line $y = x$
 (2) reflection in the origin, or rotation about the origin through 180° (clock- or anti-clockwise)
 (3) translation 2 units to the right & 5 units down
 (4) rotation 90° anti-clockwise
 (5) reflection about the line $y = -x$
 (6) reflection in the x -axis
 (7) rotation 90° clockwise (8) reflection in the y -axis



- 3.1 (1) 3 < (2) 13 <
 3.2 (a) $(x; y) \rightarrow (x+3; y+4)$ <
 (b) $(x; y) \rightarrow (x+3; y-3)$ <
 (c) $(x; y) \rightarrow (x; y-5)$ <

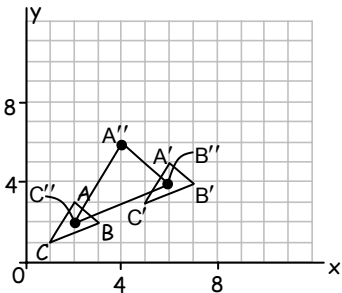
4.1 Translation, 4 units right and 2 units up. <

4.2 $A''(4; 6) ; B''(6; 4) ; C''(2; 2)$ - see Grid <

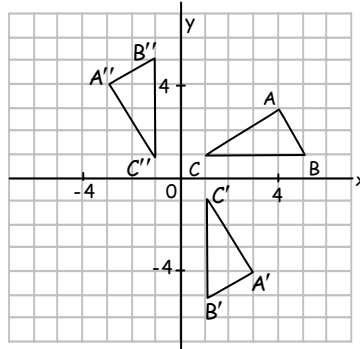
... $A(2; 3) ; B(3; 2) ; C(1; 1)$

4.3 $(x; y) \rightarrow (x+4; y+2) \rightarrow (2(x+4); 2(y+2))$ <

- 4.4 (1) $\triangle ABC \cong \triangle A'B'C'$ <
 (2) $\triangle ABC \sim \triangle A''B''C''$ <

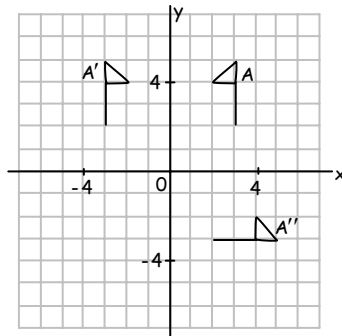


5. $A(4; 3) ; B(5; 1) ; C(1; 1)$
 5.1 $A'(3; -4) ; B'(1; -5) ; C'(1; -1)$ - see Grid <
 5.2 $A''(-3; 4) ; B''(-1; 5) ; C''(-1; 1)$ - see Grid <
 5.3 Rotation about the origin, 90° anti-clockwise <
 5.4 Rotation about the origin, 90° clockwise <



- 6.1 Reflection in the y -axis: $(x; y) \rightarrow (-x; y)$ <
 6.2 Translation 6 units right: $(x; y) \rightarrow (x+6; y)$ <

7.1 & 7.2



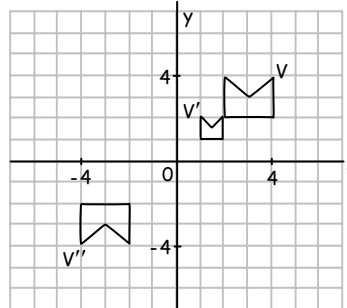
7.3 Rotation 90° clockwise about the origin <

8.1.1 See grid

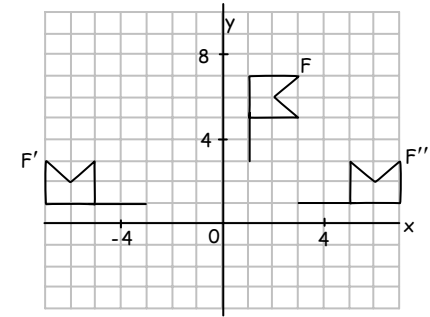
8.1.2 $\frac{P}{4}$ square units <

8.2.1 $(x; y) \rightarrow (-x; -y)$ <

8.2.2 See grid



9.1 & 9.2

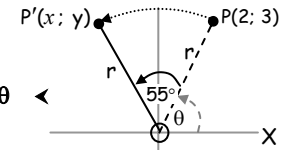


NB: Study pg. 1.36 (3rd column) to understand Q10.

10. Let $\hat{POX} = \theta$

$$\frac{2}{r} = \cos \theta \quad \& \quad \frac{3}{r} = \sin \theta$$

$$\therefore 2 = r \cos \theta \quad \& \quad 3 = r \sin \theta \quad \<$$



10.1 Let point P' be $(x; y)$

$$\frac{x}{r} = \cos(\theta + 55^\circ) \quad \& \quad \frac{y}{r} = \sin(\theta + 55^\circ)$$

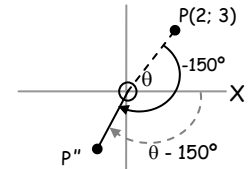
$$\therefore x = r \cos(\theta + 55^\circ) \quad \& \quad y = r \sin(\theta + 55^\circ)$$

$$\begin{aligned} &= r[\cos \theta \cos 55^\circ - \sin \theta \sin 55^\circ] = r[\sin \theta \cos 55^\circ + \cos \theta \sin 55^\circ] \\ &= \underline{r \cos \theta \cos 55^\circ} - \underline{r \sin \theta \sin 55^\circ} = \underline{r \sin \theta \cos 55^\circ} + \underline{r \cos \theta \sin 55^\circ} \\ &= 2 \cos 55^\circ - 3 \sin 55^\circ = 3 \cos 55^\circ + 2 \sin 55^\circ \\ &\approx -1,31 \qquad \qquad \qquad \approx 3,36 \end{aligned}$$

$$\therefore P'(-1,31 ; 3,36) \quad \<$$

10.2 **NB:** $\hat{XOP''} = \theta - 150^\circ!$

$$\therefore \text{Coordinates of } P''(x; y): \\ (r \cos(\theta - 150^\circ); r \sin(\theta - 150^\circ))$$



$$\text{We have: } \frac{2}{r} = \cos \theta \quad \& \quad \frac{3}{r} = \sin \theta \quad \dots \text{ (see above)}$$

$$\therefore 2 = r \cos \theta \quad \& \quad 3 = r \sin \theta$$

$$\begin{aligned} x &= r \cos(\theta - 150^\circ) \quad \& \quad y &= r \sin(\theta - 150^\circ) \\ &= r[\cos \theta \cos 150^\circ + \sin \theta \sin 150^\circ] = r[\sin \theta \cos 150^\circ - \cos \theta \sin 150^\circ] \\ &= \underline{r \cos \theta \cos 150^\circ} + \underline{r \sin \theta \sin 150^\circ} = \underline{r \sin \theta \cos 150^\circ} - \underline{r \cos \theta \sin 150^\circ} \\ &= 2 \cos 150^\circ + 3 \sin 150^\circ = 3 \cos 150^\circ - 2 \sin 150^\circ \\ &= 2\left(-\frac{\sqrt{3}}{2}\right) + 3\left(\frac{1}{2}\right) = 3\left(-\frac{\sqrt{3}}{2}\right) - 2\left(\frac{1}{2}\right) \\ &= -\sqrt{3} + \frac{3}{2} = -\frac{3\sqrt{3}}{2} - 1 \end{aligned}$$

$$\therefore P''\left(-\sqrt{3} + \frac{3}{2}; -\frac{3\sqrt{3}}{2} - 1\right) \quad \<$$

